How groups grow

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Abstract

Let G be a finitely generated group with a finite generating set S. For an element $g \in G$ let $|g|_S = \min\{n \mid g = s_1 \dots s_n, s_i \in S^{\pm}\}$ be the *length* of g with respect to S. The *growth function* of G (with respect to S) is the function defined as $\gamma_{(G,S)}(n) = |\{g \in G \mid |g|_S \leq n\}|$. The asymptotic behaviour of this function is independent of the generating set S and is an invariant of the group G.

The study of growth functions of groups started in 1970's in relation with Riemannian geometry and quickly became a central notion in geometric group theory. Natural questions in this context are regarding the relation between the structure of a group and the behaviour of its growth function. Moreover the study of this invariant has close connections with many other notions such as amenability and random walks on groups.

This talk will be a survey of the history and recent developments regarding the growth of groups. We will touch upon important milestones such as the works of Milnor and Wolf regarding solvable groups and Gromov's celebrated theorem regarding groups with polynomial growth. The main focus will be on groups whose growth functions have intermediate growth. Whether such groups exists was asked in 1970's by Milnor and the first examples were constructed by Grigorchuk in 1982.

The talk is intended to be informative to a broad audience.